MATH 111 Practice Test1

1. Construct a product table for the experiment of rolling two tetrahedral dice (4-sided) then answer these questions:

a. In how many ways can a sum of 5 be rolled?

b. In how many ways can a sum of 8 be rolled?

c. In how many ways can a sum of 13 be rolled?

2. Construct a tree diagram showing all possible results when three fair coins are tossed then answer these questions:

a. List the ways of getting no more than two heads

b. List the ways of getting exactly two heads.

3. A group of 8 strangers sat in a circle, and each got acquainted only with the person to the left and the person to the right. Then all 8 stood up and each one shook hands with each of the others who was still a stranger. How many handshakes occurred? Use a product table to confirm your answer.

4. Evaluate each expression:

a. $6! =$

b. $\frac{10!}{2!8!} =$

c. $P(9,4) =$

d. $C(10,0) =$

5. Counting numbers are to be formed using only digits from the set {0, 1, 2, 3}

a. Determine the number of 4-digit numbers that can be formed (note: 0124 and 0003 are not 4-digit numbers)

b. Determine the number of 4-digit numbers that have one pair of adjacent 0s and no other repeated digits

6. A postman arrives at the last building on his route with 4 rain soaked letters with the names smudged beyond recognition.. There are 4 mail boxes inside the foyer. The names on the mail boxes are Jones, Johns, Smith and Whatbigteethyouhavemydear.

a. In how many ways could he distribute the 4 letters if he has no idea what names are on the letters.

b. In how many ways could he distribute the 4 letters if he can tell that the last name on two of the letters begins with a J and the last name on one of the letters is very long.

7. Andy, Betty, Clyde, Dawn, Evan, and Felicia have reserved six seats in a row at the theater starting with an aisle seat. In how many ways can they arrange themselves if Betty and Dawn insist on sitting side-by-side but there are no other restrictions?

8. In 5-card poker, a straight is five cards of consecutive denominations (we will allow them to be

of the same suit).

a. In how many ways can you get a straight beginning with Ace and ending with 5?

b. In how many ways can you get any straight (including a straight flush (a straight with all cards of the same suit) and a royal flush (an Ace high straight with all cards of the same suit))?

9. Each team in an eight-team basketball league is scheduled to play each other team once. How many games will be played altogether?

10. A music class of six girls and four boys is having a recital. If each member is to perform once, how many ways can the program be arranged if:

a. A girl must perform first and a boy must perform last?

b. The first and last performers must be girls.

11. A five person committee must be chosen from a group of 6 Republicans and 5 Democrats. If two persons must be chosen from the Republicans and three persons from the Democrats, how many possible committees are there?

12. Suppose 10 fair coins are tossed. Find the number of ways of obtaining exactly four heads.

13. Write the first 7 rows (row 0 thru row 6) of Pascal’s Triangle.

14. Use Pascal’s Triangle to determine the number of possible outcomes for tossing a fair coin 6 times: 0-heads? 1-head? 2-heads? 3-heads? 4-heads? 5-heads? 6-heads?

15. Suppose 12 fair coins are tossed. Find the number of ways of obtaining at least 2 heads.

16. There are 2,598,960 possible 5-card poker hands. How many of them contain at least one face card (king, queen, or jack)? You may leave your answer in P(n,r) and/or C(n,r) form.

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The answers?

1.

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

a. 4 b. 1 c. 0

2. hhh, hht, hth, thh, tth, tht, htt, ttt

a. 0h + 1h + 2h = 1 + 3 + 3 = 7

b. 2h = 3

3.

	1	2	3	4	5	6	7	8
1			X	X	X	X	X	
2				X	X	X	X	X
3					X	X	X	X
4						X	X	X
5							X	X
6								X
7								
8								

20 handshakes.

4. a. 720 b. 45 c. 3024 d. 1

5. a. the leftmost digit cannot be 0, so $n_1 = 3$

the next digit can be any of the 4 digits, so $n_2 = 4$

the next digit can be any of 4 digits, so $n_3 = 4$

the rightmost digit can be any of the 4 digits, so $n_4 = 4$ choices.

Thus we have: 3 x 4 x 4 x 4 = 192

b. We could have a00b or ab00 as our two patterns, so we have $n_1 = 2$

there are 3 choices for digit a, 1, 2, or 3 so $n_2 = 3$

Digit b cannot be a zero nor can it duplicate digit a, so we have $n_3 = 2$

So we have: 2 x 3 x 2 = 12 possible 4-digit numbers with a pair of adjacent zeros and no other repeated digits.

6. a. The first letter could be inserted into any of the 4 mailboxes, so $n_1 = 4$

The first letter could be inserted into any of the 4 mailboxes, so $n_2 = 4$

The first letter could be inserted into any of the 4 mailboxes, so $n_3 = 4$

The first letter could be inserted into any of the 4 mailboxes, so $n_4 = 4$

Thus we have: 4 x 4 x 4 x 4 = 256 choices

b. The last name on the first letter begins with a J and there are two names beginning with J, so $n_1 = 2$

The last name on another letter begins with a J, and there are two names beginning with J, so $n_2 = 2$

There is a letter with a very long last name and there is one name that is very long, so $n_3 = 1$

The last letter could be inserted into any of the 4 mailboxes, so $n_4 = 4$

Thus we have: 2 x 2 x 1 x 4 = 16

7. Betty and Dawn insist on sitting together so we could have BDPPPP, PBDPPP, PPBDPP, PPPBDP, PPPBD so $n_1 = 5$

Betty could sit to the left or the right of Dawn, so we have $n_2 = 2$

The remaining 4 people could sit in any order and since order is important, we have $n_3 = P(4,4)$

Thus we have: $5 \times 2 \times P(4, 4) = 240$

8. a. Place the 4 aces in one pile, the 4 twos in another pile, the 4 threes in a pile, the 4 fours in a pile and the 4 fives in a pile.

You can use any of the 4 Aces, so $n_1 = 4$

You can use any of the 4 twos, so $n_2 = 4$

You can use any of the 4 threes, so $n_3 = 4$

You can use any of the 4 fours, so $n_4 = 4$

You can use any of the 4 fives, so $n_5 = 4$

Thus we have: $4 \times 4 \times 4 \times 4 \times 4 = 1024$

b. There are 10 choices for our type of straight:

Ace, Two, Three, Four, Five

Two, Three, Four, Five, Six

Three, Four, Five, Six, Seven

Four, Five, Six, Seven, Eight

Five, Six, Seven, Eight, Nine

Six, Seven, Eight, Nine, Ten

Seven, Eight, Nine, Ten, Jack

Eight, Nine, Ten, Jack, Queen

Nine, Ten, Jack, Queen, King

Ten, Jack, Queen, King, Ace

So $n_1 = 10$

Now we can work with any of the ten types of straight, suppose we work with Seven, Eight, Nine, Ten, Jack

There are 4 choices for which seven we'll have, so $n_2 = 4$

There are 4 choices for which eight we'll have, so $n_3 = 4$

There are 4 choices for which nine we'll have, so $n_4 = 4$

There are 4 choices for which ten we'll have, so $n_5 = 4$

There are 4 choices for which jack we'll have, so $n_6 = 4$

Thus we have: 10 x 4 x 4 x 4 x 4 x 4 = 10240

Notice that we did not attempt to eliminate the straights that are also flushes.

9. In order to have a basketball game, we need two of the teams to play against each other so we have: C(8, 2) = 28

Order is not important since we don't care where they play the game.

10. Since a girl must play first and there are 6 girls, $n_1 = 6$

Next we need to select a boy who will play last. There are 4 boys, so $n_2 = 4$

Now there are 5 girls plus 3 boys who need to be selected so we have n_3 = P(8, 8)

Order is important, so we use permutations

Thus we have: 6 x 4 x P(8, 8) = 967,680

b. Since a girl must perform first and there are 6 girls, so $n_1 = 6$

Now we need to select a girl who will play last. There are 5 girls to choose from, so $n_2 = 5$

Now we need to assign places for the remaining 8 students, so n_3 = P(8)

Thus we have: 6 x 5 x P(8, 8) = 1,209,600

11. Order is not important so when we choose 2 of the Republicans, we have: n_1 = C(6,2)

Order is not important so when we choose the Democrats we have: n_2 = C(5,3)

Thus we have: C(6, 2) x C(5, 3) = 150

12. It turns out that we can use C(10, 4) to determine the number of ways of getting exactly 4 heads when we toss 10 coins, so we have: C(10, 4) = 210

13.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

14. Using the table above:

0-head 1 way

1-head 6 ways

2-heads 15 ways

3-heads 20 ways

4-heads 15 ways

5-heads 6 ways

6-heads 1 ways

15. There are 2 possible outcomes for each toss of the coin, so the total possible outcomes are:

$$2^{12} = 4096$$

Another way to describe the possible outcomes is, "the ways we could have 0-heads **plus** the ways we could have 1-head **plus** the ways we could have 2- heads, … **plus** the number of ways we could have 12 heads," or:

$$C(12, 0) + C(12, 1) + C(12, 2) + . . . + C(12, 12) = 4096$$

Only 0-heads and 1-head fail to have "at least 2 heads, so our problem can be stated as:

$$2h + 3h + . . . + 12h = 4096 – 0h – 1h$$

$$2h + 3h + … + 12h = 4096 – C(12, 0) – C(12, 1)$$

$$= 4096 – 1 – 12$$

$$= 4083$$

16. We could partition the deck of cards into hands with exactly no face cards **plus** the hands with exactly 1-face card **plus** the hands with exactly 2-face cards **plus** the hands with exactly 3-face cards **plus** the hands with exactly 4-face cards **plus** the hands with exactly 5-face cards.

0-face cards + 1-face card + 2-face cares + 3-face cards + 4-face cards + 5-face cards = 2,598,960

Only those hands with 0-face cards would fail to satisfy our condition. There are 12 face cards and 40 non-face cards in the deck. To end up with no face cards, we must choose none of the face cards and 5 of the non-face cards or, C(40, 5)C(12, 0)

So we'll subtract the hands that have no face cards from the total possible hands to get:

$$1f + 2f + 3f + 4f + 5f = 2,598,960 - C(40,5)C(12,0)$$

$$1f + 2f + 3f + 4f + 5f = 2,598,960 - 658,008$$

$$1f + 2f + 3f + 4f + 5f = 1,940,952$$