

MATH 115 T3 Practice

1. Calculate (a) the mean, (b) median, and (c) the mode or modes:

12.3, 45.6, 78.9, 1.2, 34.5, 67.8, 90.1

2. Calculate (a) the mean, (b) median, and (c) the mode or modes:

x_i	f_i	$x_i \cdot f_i$	Cum. f_i
2	5		
4	3		
6	8		
8	4		

3. Determine the (a) range, and (b) standard deviation for this sample (Note mean = 31):

34, 22, 41, 30, 28

4. Use the table to determine the standard deviation for the sample distribution listed below:

x_i	f_i	$x_i \cdot f_i$	$(x_i - \text{mean})$	$(x_i - \text{mean})^2$	$f_i \cdot (x_i - \text{mean})^2$
1	7				
2	4				
3	8				
4	11				

5. Find the percent of the total area under the normal curve between these values of z

(a) $z = 0.41$ and $z = 2.83$

(b) Find the total area between $z = -1.15$ and $z = 2.00$

(c) Find the total area to the left of $z = -.58$

6. Two hundred MATH 115 students measured the number of hours of homework they did during the Fall '94 semester. The results are found to closely approximate a normal curve, with mean 14 hours and standard deviation 2 hours.

(a) more than 16 hours

(b) between 12 and 16 hours

7. My light bulbs have an average life of 500 hr, with a standard deviation of 100 hr. The length of life of the bulb can be closely approximated by a normal curve. If you install 5,000 such bulbs, find the total number that can be expected to last the following amounts of time:

(a) less than 740 hr

(b) between 350 and 700 hours

(c) less than 400 hours

8. a. For a normal distribution, find a value of z such that 10% of the total area is to the right of z

b. Determine the value of z such that 50% of the area under the normal curve is between z and $-z$.

c. On a test given to 500 calculus students, the mean score was 78 with a standard deviation of 9. If the scores are approximately normally distributed, determine the score such that 90% of the scores are lower than that score.

9. Use a normal approximation to the binomial distribution for each of the following:

(a) If 20 fair coins are tossed, find the probability of exactly 5 heads.

(b) If 20 fair coins are tossed, what is the probability of tossing at least 14 heads.

10. According to Chebyshev's Theorem, what fraction of the scores lie within 1.75 standard deviations of the mean? (round off to the nearest tenth of a percent)_

1a. mean = $\frac{330.4}{7} = 47.2$

1b. median = 45.6 1.2, 12.3, 34.5, 45.6, 67.8, 78.9, 90.1

1c. mode = none

2. Calculate (a) the mean, (b) median, and (c) the mode or modes:

x_i	f_i	$x_i \cdot f_i$	Cum. f_i
2	5	10	5
4	3	12	8
6	8	48	16
8	4	32	20

a) mean = $102/20 = 5.1$

b) median = 6

c) mode = 6

3. a. range = $41 - 22 = 19$

b. $s = \sqrt{\frac{(34-31)^2 + (22-31)^2 + (41-31)^2 + (30-31)^2 + (28-31)^2}{5-1}} = \sqrt{\frac{9+81+100+1+9}{4}} = 7.07$

4. Use the table to determine the standard deviation for the sample distribution listed below:

x_i	f_i	$x_i \cdot f_i$	$(x_i - \text{mean})$	$(x_i - \text{mean})^2$	$f_i \cdot (x_i - \text{mean})^2$
1	7	7	$(1-2.77) = -1.77$	$(-1.77)^2 = 3.13$	$7(3.13) = 21.91$
2	4	8	$(2-2.77) = -.77$	$(-.77)^2 = .59$	$4(.59) = 2.36$
3	8	24	$(3-2.77) = .23$	$(.23)^2 = .05$	$8(.05) = .40$
4	11	44	$(4-2.77) = 1.23$	$(1.23)^2 = 1.51$	$11(1.51) = 16.61$

$$\text{Mean} = \frac{83}{30} = 2.77$$

$$s = \sqrt{\frac{21.91+2.36+.40+16.61}{30-1}} = \sqrt{\frac{41.28}{29}} = \sqrt{1.42} = 1.19$$

5. a.

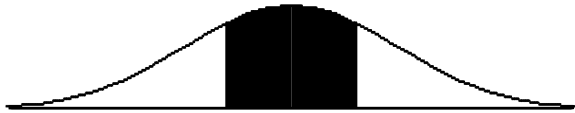


For $z = 0.41$, $A = 0.159$

For $z = 2.83$, $A = 0.498$

This is a detached region, so we subtract: $A = 0.498 - 0.159 = 0.339$

5. b.

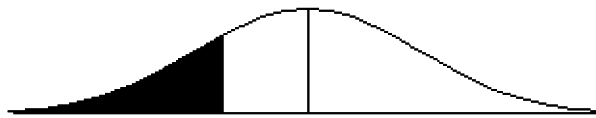


For $z = -1.15$, $A = 0.375$

For $z = 2.00$, $A = 0.477$

So $A = 0.375 + 0.477 = 0.852$

5. c.

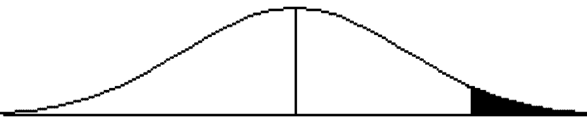


For $z = 0$, $A = 0.500$

For $z = -.58$, $A = 0.219$

This is a detached region so we subtract: $A = 0.500 - 0.219 = 0.281$

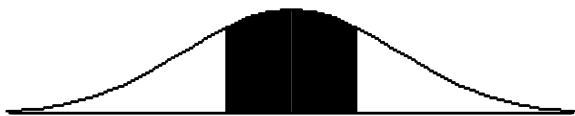
6. a.



$$z = \frac{16 - 14}{2} = 1 \quad \text{So } A = 0.341$$

16 is to the right of the mean, 14, so we subtract: $A = 0.500 - 0.341 = 0.159$

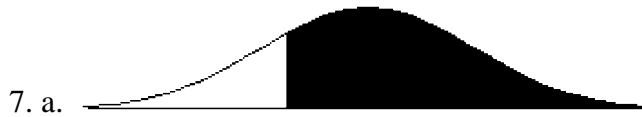
6. b.



$$z = \frac{12 - 14}{2} = -1 \text{ so } A = 0.341$$

$$z = \frac{16 - 14}{2} = 1 \text{ so } A = 0.341$$

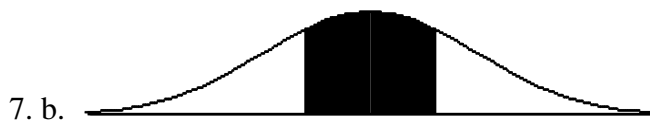
12 is to the left of the mean, 14, and 16 is to the right of the mean, so total Area = $0.341 + 0.341 = 0.682$



$$z = \frac{740 - 500}{100} = 2.4 \text{ so } A = 0.492$$

740 is to the right of the mean, 500, so we add: total area = $0.500 + 0.492 = 0.992$

Number of bulbs in this region: $0.992 \times 5000 = 4,960$



$$z = \frac{350 - 500}{100} = -1.5 \text{ so } A = 0.433$$

$$z = \frac{700 - 500}{100} = 2 \text{ so } A = 0.477$$

Total area = $0.433 + 0.477 = 0.910$

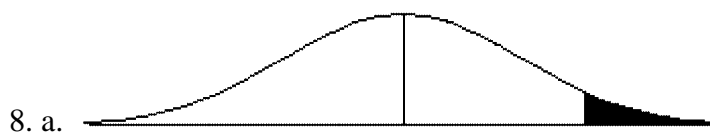
Number of bulbs in this sample: $0.910 \times 5000 = 4,550$



$$z = \frac{400 - 500}{100} = -1 \text{ so } A = 0.341$$

Total area = $0.500 - 0.341 = 0.159$

Number of bulbs in sample: $0.159 \times 5000 = 795$



Area in the shaded region is 10%, so we'll use $A = .40$ and determine z .

For $A = .400$, $z = 1.28$



Since 50% of the area is between z and $-z$, each of the smaller shaded regions must have 25% of the region. For $A = .250$, $z = .67$
Thus $z = .67$ and $-z = -.67$



We want 90% in the shaded region. 50% of the scores are to the left of the mean, so 40% must be in the shaded region to the right of the mean (hmmm, not to scale).
For $A = .40$, $z = 1.28$, so we have:

$$1.28 = \frac{x - 78}{9}$$

$$9(1.28) = x - 78$$

$$11.52 = x - 78$$

$$11.52 + 78 = x$$

$$89.52 = x$$

Use 90 as x .

9. a.

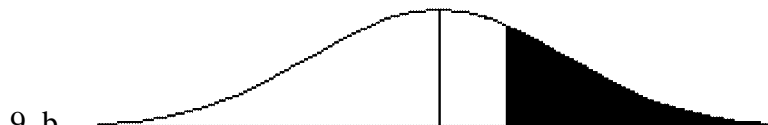
$$\text{mean} = (20)(.5) = 10$$

$$\text{Standard deviation} = \sqrt{(20)(.5)(.5)} = \sqrt{5} = 2.24$$

$$Z = \frac{4.5 - 10}{2.24} = \frac{5.5}{2.24} = 2.46 \text{ so } A = 0.493$$

$$Z = \frac{5.5 - 10}{2.24} = \frac{4.5}{2.24} = 2.01 \text{ so } A = 0.478$$

$$P(\text{exactly } 5) = .493 - .478 = .015$$



mean = 10 Standard deviation = 2.24

$$z = \frac{13.5 - 10}{2.24} = \frac{3.5}{2.24} = 1.56 \text{ so } A = 0.441$$

$$P(\text{at least } 14 \text{ heads}) = .500 - .441 = .059$$

$$10. \quad 1 - \frac{1}{k^2} = 1 - \frac{1}{1.75^2} = 1 - \frac{1}{3.06} = 1 - .327 = .673 \text{ or } 67.3\%$$