MATH 111e F '94 T2 Practice

1. Assume that two fair dice are rolled. Determine the probability of rolling the following sums:

a. 3

b. 5

2. Three married couples arrange themselves randomly in six consecutive seats in a row. Find these probabilities:

a. Each man will sit immediately to the left of a woman.

b. The three women will sit in three adjacent seats and the men will sit in three adjacent seats.

3. In 5-card poker, find the probability of being dealt each of the following (you may leave your answer in C(n,r) and/or P(n,r) form):

a. a flush (all 5 cards of the same suit)

b. a pair

4. For the experiment of drawing a single card from a standard 52-card deck, (i) find the probability, and (ii) the odds in favor, of each of the following events:

a. king or a face card

b. king or queen

c. An ace, given it is a face card

5. For the experiment of rolling two tetrahedral (four sided) dice and noting the sum of the pips, find these probabilities:

a. even or multiple of 3

b. less than 3 or multiple of 2

6. One hundred college students, categorized by sex, indicated which science courses they were taking. The results are summarized below:

	Physics	Chemistry	Biology	Total
Male	15	27	32	74
Female	10	8	8	26
Total	25	35	40	100

One student is selected at random, find these probabilities:

a. Male

b. Female and taking Biology

c. male, given that the student is taking Chemistry

d. P(physics | Female)

7. Two cards are dealt successively, with replacement, from a standard 52-card deck. Find these probabilities:

a. one jack and one queen

b. a jack, then a queen

8. For n repeated independent trials, with constant probability of success p for all trials, find the probability of exactly x successes for n = 10, p = .7 and x = 3

9. Robert is taking a 10 question multiple choice test. Each question has 5 choices, with only 1 correct answer. If Robert guesses on all 10 questions, determine these probabilities:

a. Robert gets exactly 2 correct answers.

b. Robert gets at least 2 correct answer.

10. In planting a playing field, a park manager must decide between planting seed or sod. If seed is used, there is a 33% chance that grass lawn will grow with one seeding and a 67% chance that it will need two seedings. If the lawn is seeded once, it will cost $60. If the lawn needs two seedings, the cost will be $400. Planting sod will cost $300 and has a 100% success rate. Which method is more cost-effective? (This might be the comparison of two expected value situations)

11. The town of Birkbeck has a Pick 4. Since they were short of ping pong balls, each of the four containers has 4 balls labeled {1, 2, 3, 4}. The players pay $2.00 and then select a 4-digit number. If you pick the correct number, you receive $50. What is the expected value of this lottery?

12. A medical test is known to be 95% accurate. If 15% of the population is known to have the disease, answer these questions:

a. A person with the disease will test positive?

b. A person will have a false negative test result?

c. A person will have a false positive test result?

The answers?

1.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

a. $P(3) = \frac{2}{36} = \frac{1}{18}$ b. $P(5) = \frac{4}{36} = \frac{1}{9}$

2a.

The only possible seating arrangement is: mwmwmw

n_1 There are 3 choices for the man on the far left

n_2 There are 3 choices for the leftmost woman

n_3 There are 2 choices for the next man

n_4 There are 2 choices for the next woman

n_5 There is 1 choice for the last man

n_6 There is 1 choice for the last woman

Possible seating arrangements with mwmwmw = (3)(3)(2)(2)(1)(1) = 36

Total possible seating arrangements P(6,6) = 6!(6-6)! = 6!0! = 720

P(a man will sit to the left of each woman) $= \frac{36}{720} = \frac{1}{20} = .05$

2. b. n_1 = possible seating arrangements: wwwmmm, mmmwww = 2

n_2 = possible arrangements of the 3 women = P(3,3) = 6

n_3 = possible arrangements of the 3 men = P(3,3) = 6

Seating arrangements with women together = (2)(6)(6) = 72

Total possible seating arrangements = P(6,6) = 720

P(women and men sit together) = $\frac{72}{720} = \frac{1}{10} = .1$

3. a. n_1 = there are 4 suits (clubs, hearts, diamonds, spades) = 4

n_2 = Take 5 of the 13 cards in one of the suits = C(13, 5)

n_3 = The total 5-card hands possible = C(52, 5)

P(any flush) = $\frac{(4)(C(13,5))}{C(52,5)}$ This includes straight flushes and royal flushes

b. First partition the 52 cards in 13 piles with cards of the same rank together (for example all aces in one pile, kings in another pile, etc.)

n_1 = select one of the piles = 13

n_2 = Take 2 cards from that pile = C(4,2)

n_3 = Select three more piles (we'll take one card from each of them) = C(12,3)

n_4 = Select a card from one of the piles from n_3 = C(4,1)

n_5 = Select a card from another of the piles from n_3 = C(4,1)

n_6 = Select a card from the last of the piles from n_3 = C(4,1)

5-card hands with exactly 1 pair = (13)(C(4,2))(C(12,3))(C(4,1))(C(4,1))(C(4,1))

n_7 = the total number of 5-card hands possible = C(52, 5)

P(Exactly one pair) = $\frac{(13)(C(4,2))(C(12,3))(C(4,1))(C(4,1))(C(4,1))}{C(52,5)}$

4a. Twelve of the 52 cards are face cards (that includes the 4 kings) so,

P(king or face) $= \frac{12}{52} = \frac{3}{13}$ The odds in favor are 3 to 10 or 3:10

b There are 4 kings and 4 queens, so

P(king or queen) $= \frac{8}{52} = \frac{2}{13}$ The odds in favor are 2 to 11 or 2:11

c. The face cards are King, Queen, and Jack so if you have a face card you cannot get an ace. Thus, P(ace | face card) $= \frac{0}{12} = 0$ Thus it is impossible to calculate the odds.

5.

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

a. Even numbers {2, 4, 6, 8} Multiples of 3 are {3, 6} Then count how many 2s, 3s, 4s, 6s and 8s are in the interior of the table:

P(even or multiple of 3) $= \frac{10}{16} = \frac{5}{8}$

b. Less than 3 = {2} Multiples of 2 = {2, 4, 6, 8} Then count the numbers of 2s, 4s, 6s, and 8s in the interior of a table:

P(less than 3 or a multiple of 2) $= \frac{8}{16} = \frac{1}{2}$

6. a

	Physics	Chemistry	Biology	Total
Male	15	27	32	74
Female	10	8	8	26
Total	25	35	40	100

a. P(male) = $\frac{74}{100}$ = 0.74

6. b.

	Physics	Chemistry	Biology	Total
Male	15	27	32	74
Female	10	8	8	26
Total	25	35	40	100

P(female and taking biology) = $\frac{8}{100}$ = 0.08

6. c.

	Physics	Chemistry	Biology	Total
Male	15	27	32	74
Female	10	8	8	26
Total	25	35	40	100

P(male | taking chem.) = $\frac{27}{35}$ = .77

6. d.

	Physics	Chemistry	Biology	Total
Male	15	27	32	74
Female	10	8	8	26
Total	25	35	40	100

P(physics | female) $= \frac{10}{26} = \frac{5}{13}$

7. a. We could get a jack on the first draw and a queen on the second draw, JQ or we could get a queen on the first draw and a jack on the second draw, QJ.

P(jack) $= \frac{4}{52}$ and P(queen) $= \frac{4}{52}$

P(1 jack and 1 queen) = P(JQ) + P(QJ) = $\frac{4}{52} \times \frac{4}{52} + \frac{4}{52} \times \frac{4}{52} = \frac{1}{169} + \frac{1}{169} = .0118$

b. P(jack then queen) = P(JQ) $= \frac{4}{52} \times \frac{4}{52} = \frac{1}{169} = .0059$

8. $C(10,3)(.7^3)(.3^7) = 120(.343)(.0002187) = .009$

9. a. There are 5 choices and he is guessing so, P(correct answer) $= \frac{1}{5} = .2$

and P(incorrect answer) $= \frac{4}{5} = .8$

P(exactly 2 correct) = $C(10,2)(.2^2)(.8^8) = (45)(.04)(.16777) = .30199$

9. b. P(at least 2 correct) = P(2 correct) + P(3 correct) +…+ P(10 correct)

or P(at least 2 correct) = 1 – P(0 correct) – P(1 correct)

$= 1 - C(10,0)(.2^{0})(.8^{10}) - C(10,1)(.2^{1})(.8^{9})$

$= 1 - .10737 - .26944$

$= .62419$

10.

	P(success)	Cost
1-seeding	.33	\$60
2-seedings	.67	\$400

Expected Value for seeding = (.33)(.60) + (.67)(400) = 19.8 + 268 = \$287.80

The expected value for using sod is \$300, and \$287.80 < \$300, so seeding is best

11. Each time a ball is chosen, there are 4 possible outcomes {1, 2, 3, 4}. Since we are drawing balls from four urns, there are $4^4 = 256$ possible outcomes.

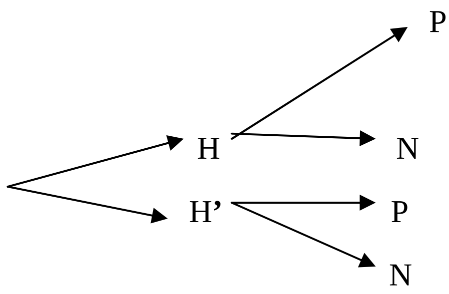
	Probability	Reward
Winning ticket	$\frac{1}{256}$	\$48
Losing ticket	$\frac{255}{256}$	-\$2.00

$$(48)\left(\frac{1}{256}\right) + (-2)\left(\frac{255}{256}\right) = \frac{48 - 510}{256} = -\$1.80$$

12. a. P(have disease and test positive) = (.15)(.95) = .1425

b. P(have disease and test negative) = (.15)(.05) = .0075

c. P(false positive) = P(don't have disease and test positive) = (.85)(.05) = .0425



HP – a person has the disease and tests positive

HN – a person has the disease and tests negative

H'P – person doesn't have disease and tests positive

H'N – person doesn't have disease and tests negative