Week 2 homework answers

1. Evaluate each of the following:

a. $8! = 8x7x6x5x4x3x2x1 = \mathbf{40{,}320}$

b. $7!/(4!3!) = \frac{7!}{4!3!} = \frac{7x6x5x4!}{3x2x1x4!} = \frac{7x6x5}{3x2x1} = \frac{210}{6} = \mathbf{35}$

c. $P(7, 5) = \frac{7!}{(7-5)!} = \frac{7!}{2!} = \frac{7x6x5x4x3x2x1}{2x1} = \mathbf{2520}$

d. $C(10, 10) = \frac{10!}{10!(10-10)!} = \frac{10!}{10!0!} = \frac{10!}{10!} = \mathbf{1}$

e. $P(9, 9) = \frac{9!}{(9-9)!} = \frac{9!}{0!} = \frac{9!}{1} = \mathbf{362{,}880}$

f. $C(6, 6) = \frac{6!}{6!(6-6)!} = \frac{6!}{6!0!} = \frac{6!}{6!} = \mathbf{1}$

2. The thirteen members of the Good Fellows club must elect a president, secretary and treasurer. If the positions are to be filled by random selection, how many possible choices are there?

Order is important since it makes a difference who is the president, who is the secretary, and who is the treasurer.

$P(13, 3) = \frac{13!}{(13-3)!} = \frac{13!}{10!} = \frac{13x12x11x10!}{10!} = \frac{13x12x11}{1} = \mathbf{1716}$

3. A bet at the racetrack is the trifecta. To bet a trifecta, you must select which horse will win, which horse will come in second, and which horse will come in third. If there are 8 horses in a race how many possible trifecta bets are there?

Order is important. It is important who wins the race, who is second, etc.

$P(8, 3) = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8x7x6x5!}{5!} = \frac{8x7x6}{1} = \mathbf{336}$

4. Seven friends arrived at a music program to find that there were only 3 seats left. How many arrangements of the seven friends are available if three will sit and the others will stand?

If where you sit is important: $P(7, 3) = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7x6x5x4!}{4!} = \frac{7x6x5}{1} = \mathbf{210}$

If just having a seat is important $C(7, 3) \frac{7!}{3!(7-3)!} = \frac{7x6x5x4!}{3x2x1x4!} = \frac{210}{6} = \mathbf{35}$

5. Nine men volunteered to pick up trash along the road, but only four men were needed. If the four will be selected at random, how many possible arrangements are there of the men who will work?

Clearly order is not important since nobody cares who was chosen first or second, etc.

$$C(9, 4)\ \frac{9!}{4!(9-4)!} = \frac{9x8x7x6x5!}{4x3x2x1x5!} = \frac{3024}{24} = \mathbf{126}$$